Lecture 11

Black Holes

Recap

Where were we?

General Relativity

Newtonian Limit

$$ds^2 = -(1+2\Phi)\,dt^2 + (1-2\Phi)\,d\vec{x}^2$$

$$|\Phi| \ll 1\,, \quad \partial_t \Phi = 0\,, \quad p \ll \rho$$

Einstein equations

$$\boxed{R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}} \xrightarrow[\mu=\nu=0]{} \boxed{\nabla^2 \Phi \simeq 4\pi G \rho}$$

$$|\vec{v}| \ll c$$

Geodesic equations

$$\boxed{\frac{dx^\nu}{d\tau} \nabla_\nu \frac{dx^\mu}{d\tau} = 0} \longrightarrow \boxed{\frac{d^2\vec{x}}{dt^2} = -\vec{\nabla}\Phi}$$

Energy-momentum tensor

Cosmological constant

$$\frac{\Lambda}{8\pi G} = \rho_{vac}$$

$$\boxed{T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta S_M}{\delta g^{\mu\nu}} - \frac{\Lambda}{8\pi G} g_{\mu\nu}}$$

General Relativity

Actions

Einstein equations

$$\boxed{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G\, T_{\mu\nu}} \longleftarrow \boxed{S = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\,(R - 2\Lambda) + S_M}$$

Geodesic equations

$$\boxed{\frac{dx^\nu}{d\tau} \nabla_\nu \frac{dx^\mu}{d\tau} = 0} \longleftarrow \boxed{S = -m \int d\tau}$$

Energy-momentum tensor

$$\boxed{T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta S_M}{\delta g^{\mu\nu}} - \frac{\Lambda}{8\pi G} g_{\mu\nu}}$$

Cosmological constant

$$\frac{\Lambda}{8\pi G} = \rho_{vac}$$

Spherical symmetry + Vacuum $(T_{\mu\nu}=0)$ $\Rightarrow$ The Schwarzschild metric

$$\boxed{ds^2 = -\left(1-\frac{R_s}{r}\right)dt^2 + \frac{dr^2}{1-\frac{R_s}{r}} + r^2\, d\Omega^2}$$

$$\boxed{R_s = 2GM}$$

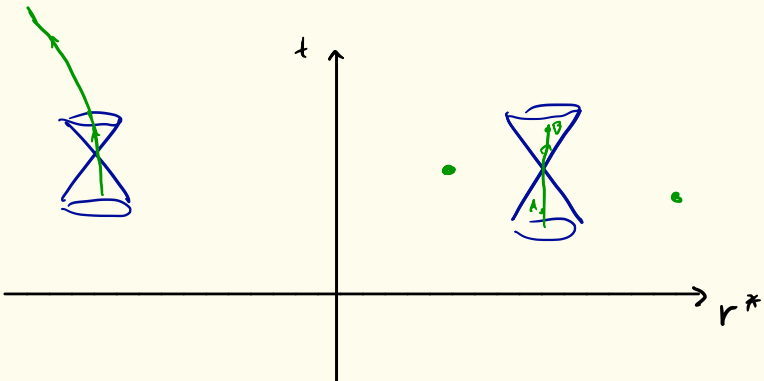
$$r^* = r + R_s \log\left(\frac{r}{R_s} - 1\right)$$ [Tortoise coordinate]

$$\boxed{ds^2 = \left(1-\frac{R_s}{r}\right)\left(-dt^2 + dr^{*2}\right) + r^2\, d\Omega^2}$$

$r = R_s$

$\Updownarrow$

$r^* = -\infty$



Plan for today

- Schwarzschild black holes
- Penrose diagrams
- Stars and black holes

Schwarzschild Black Holes

$$\boxed{ds^2 = \left(1-\frac{R_s}{r}\right)\left(-dt^2 + dr^{*2}\right) + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)}$$

Consider an observer falling radially into a BH

$$U^\mu = \left(\frac{dt}{d\tau}, \frac{dr^*}{d\tau}, 0, 0\right) \qquad E = -\underset{\substack{\uparrow \\ \partial_t = (1,0,0,0)}}{k^\mu} U_\mu = \left(1-\frac{R_s}{r}\right)\frac{dt}{d\tau}$$

$$-1 = g_{\mu\nu} U^\mu U^\nu = \left(1-\frac{R_s}{r}\right)\left[-\left(\frac{dt}{d\tau}\right)^2 + \left(\frac{dr^*}{d\tau}\right)^2\right] \Rightarrow \left(\frac{dr^*}{d\tau}\right)^2 = -\frac{1}{1-\frac{R_s}{r}} + \frac{E^2}{\left(1-\frac{R_s}{r}\right)^2}$$

Notice that $r - R_s \approx R_s\, e^{r^*/R_s}$ for $r - R_s \ll R_s$. Therefore,

$$\left(\frac{dr^*}{d\tau}\right)^2 \approx -e^{-r^*/R_s} + R^2 e^{-2r^*/R_s} \underset{r^* \to -\infty}{\approx} E^2 e^{-2r^*/R_s}$$

$$\Rightarrow \int_{\tau_1}^{\tau_H} d\tau = -\int_{r_1^*}^{-\infty} dr^* \frac{1}{E} e^{r^*/R_s} \iff \tau_H - \tau_1 = \frac{1}{E} e^{r_1^*/R_s} \quad \text{finite !}$$

$\Rightarrow$ observer falls to $r^* = -\infty \iff r = R_s$ in finite proper time.

Schwarzschild Black Holes

Now introduce null coordinates : $\begin{cases} v = t + r^* \\ u = t - r^* \end{cases}$

$$\Rightarrow \quad ds^2 = -\left(1 - \frac{R_s}{r}\right) du\, dv + r^2 d\Omega^2$$

where r is defined by $\frac{1}{2}(v-u) = r + R_s \log\left(\frac{r}{R_s} - 1\right)$

Next, change to $\begin{cases} v' = e^{v/2R_s} \\ u' = -e^{-u/2R_s} \end{cases}$ $\quad \begin{matrix} v \in \mathbb{R} \iff v' > 0 \\ u \in \mathbb{R} \iff u' < 0 \end{matrix}$

$$dv'du' = \frac{dv\, du}{4R_s^2} e^{\frac{v-u}{2R_s}} = \frac{dv\, du}{4R_s^2} e^{r/R_s}\left(\frac{r}{R_s} - 1\right)$$

$$\Rightarrow \quad \boxed{ds^2 = \frac{4R_s^3}{r} e^{-r/R_s} dv'du' + r^2 d\Omega^2}$$

Schwarzschild Black Holes

$$\boxed{ds^2 = \frac{4R_s^3}{r} e^{-r/R_s} \, dv' \, du' + r^2 d\Omega^2}$$

Finally, introduce Kruskal coordinates

$$T = \frac{v' + u'}{2} \qquad R = \frac{v' - u'}{2}$$

to obtain

$$\boxed{ds^2 = \frac{4R_s^3}{r} e^{-r/R_s} \left(-dT^2 + dR^2 \right) + r^2 d\Omega^2}$$

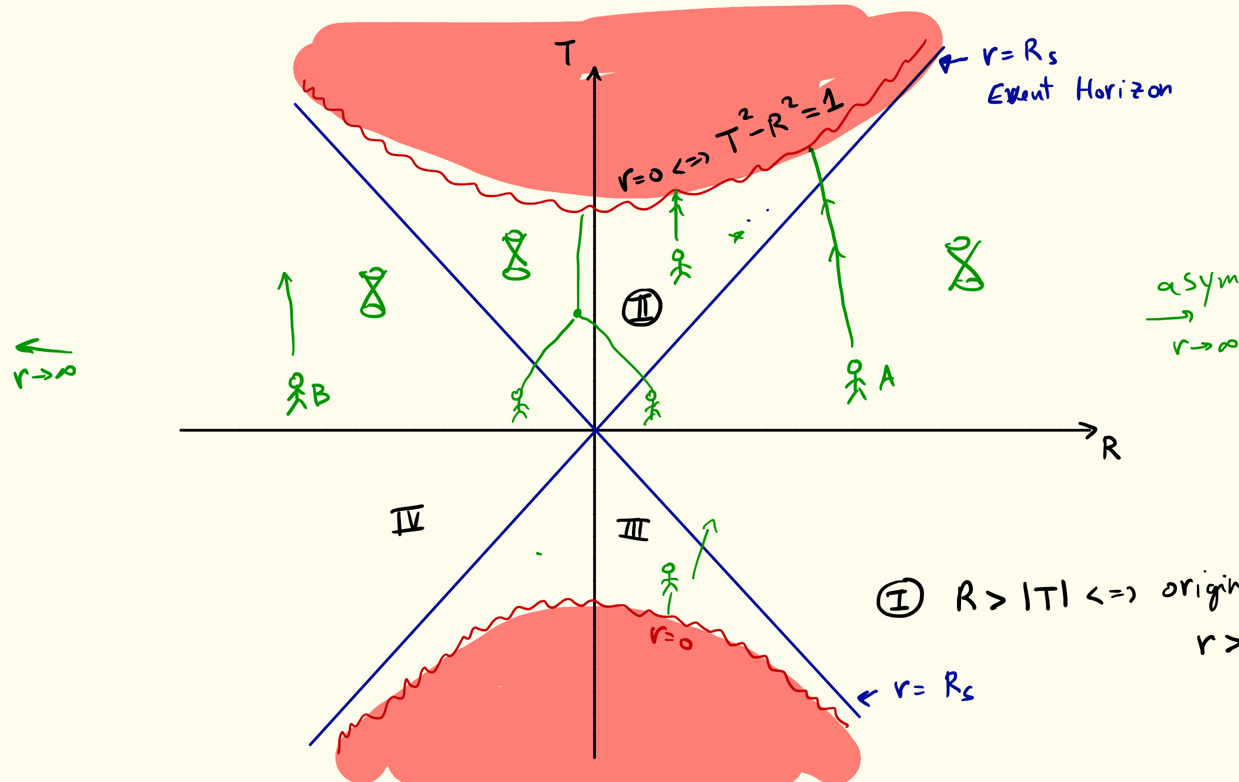
where r is defined by

$$T^2 - R^2 = \left(1 - \frac{r}{R_s}\right) e^{r/R_s}$$

The Kruskal diagram

$$\boxed{ds^2 = \frac{4R_s^3}{r} e^{-r/R_s} \left(-dT^2 + dR^2\right) + r^2 d\Omega^2}$$

$$T^2 - R^2 = \left(1 - \frac{r}{R_s}\right) e^{r/R_s}$$



I $R > |T| \Leftrightarrow$ original region

$r > R_s,\ t \in \mathbb{R}$

$T=0$: spatial surface with metric

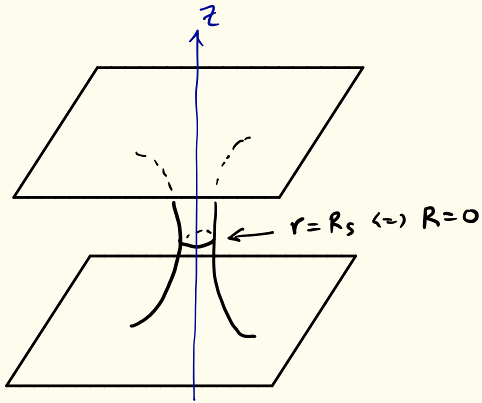
$$ds^2 = \underbrace{\frac{4R_s^3}{r} e^{-r/R_s} dR^2}_{\frac{dr^2}{1-\frac{R_s}{r}}} + r^2 d\Omega^2 \quad , \quad R^2 = \left(\frac{r}{R_s} - 1\right) e^{r/R_s}$$

Exercise : Find an isometric embedding of this surface in $\mathbb{R}^4$

$$ds^2_{\mathbb{R}^4} = dz^2 + dr^2 + r^2 d\Omega^2$$

Find $z(r)$ such that

$$dz^2 + dr^2 = (z'(r)^2 + 1) dr^2 = \frac{dr^2}{1-\frac{R_s}{r}}$$



"wormhole"

Penrose diagrams

In order to picture the entire spacetime, it is convenient to:

- use coordinates that take values in a finite interval;
- keep light cones at 45°.

Example: Minkowski spacetime

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2$$

$$\begin{cases} u = t - r \\ v = t + r \end{cases} \Rightarrow \qquad ds^2 = -du\,dv + \left(\frac{v-u}{2}\right)^2 d\Omega^2$$

$$\begin{cases} U = \arctan u \\ V = \arctan v \end{cases} \underset{\text{check!}}{\Longrightarrow} \qquad ds^2 = \frac{1}{4\cos^2 U \cos^2 V}\left[-4\,dU\,dV + \sin^2(U-V)\,d\Omega^2\right]$$

$$-\frac{\pi}{2} < U \leq V < \frac{\pi}{2}$$

Finally,

$$\begin{cases} T = V + U \\ R = V - U \end{cases} \Rightarrow \quad 0 \le R < \pi \,, \quad |T| + R < \pi$$

$\Rightarrow$
check

$$ds^2 = \frac{1}{\omega^2(T,R)} \left[-dT^2 + dR^2 + \sin^2 R \, d\Omega^2 \right]$$

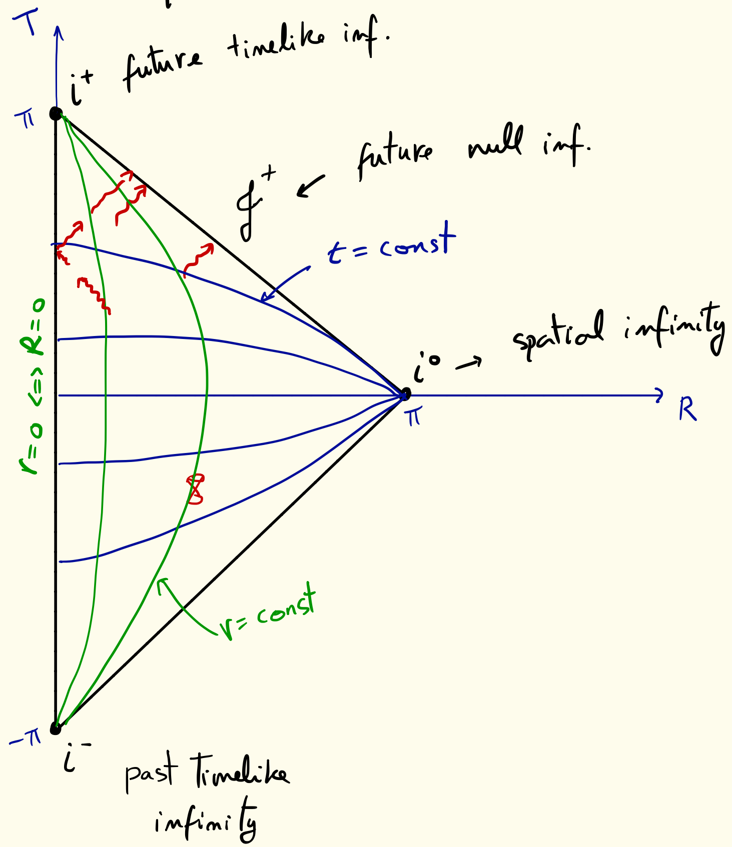
$\omega = \cos T + \cos R$

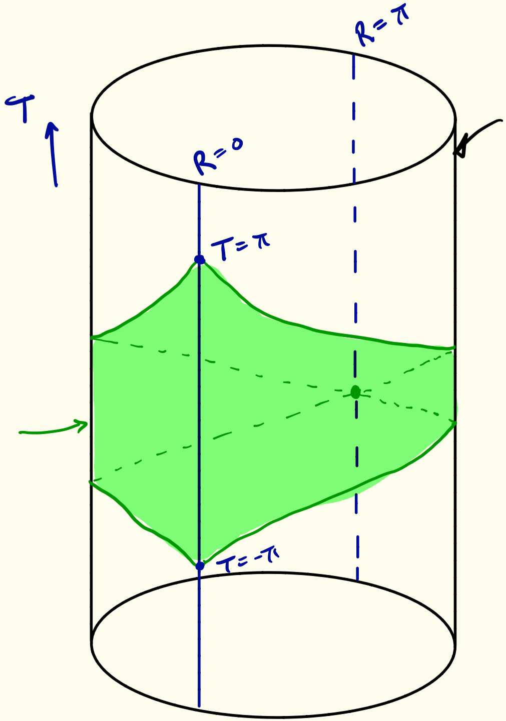
Let us define the metric

$$d\tilde{s}^2 \equiv \omega^2(T,R)\, ds^2 = \underbrace{-dT^2 + dR^2 + \sin^2 R \, d\Omega^2}_{\text{metric on } \underset{T}{\mathbb{R}} \times \underset{\text{3-sphere}}{S^3}}$$

$\rightarrow$ ds^2 and $d\tilde{s}^2$ have the same light cones!

Penrose diagram of Minkowski spacetime





$\mathbb{R} \times S^3$ = Einstein static Universe

$0 \leq R \leq \pi$

$|T| + R \leq \pi$

conformal <u>compactification</u> of Minkowski spacetime

Schwarzschild Black Holes

$$\boxed{ds^2 = \frac{4R_s^3}{r} e^{-r/R_s} \, dv' du' + r^2 d\Omega^2}$$

$u', v' \in \mathbb{R}$ $\qquad\qquad v'u' = -\left(\frac{r}{R_s} - 1\right) e^{r/R_s}$

Define

$$\begin{cases} v'' = \arctan v' \\ u'' = \arctan u' \end{cases}$$

Notice

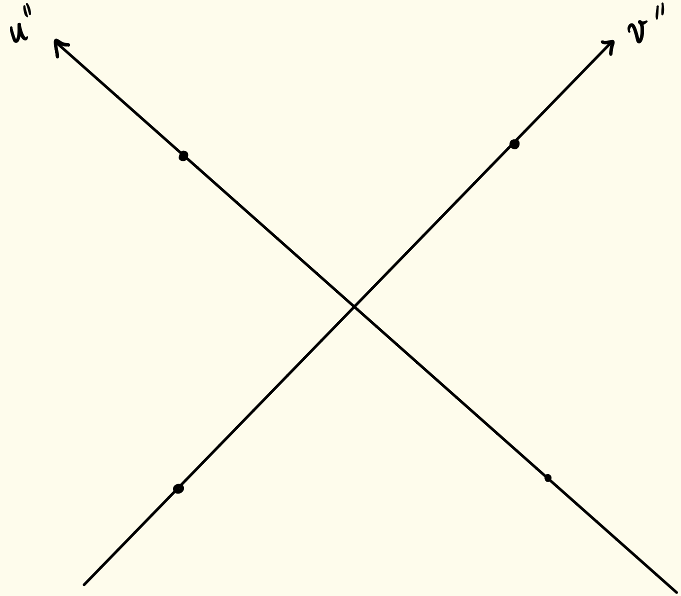
$$\tan(v''+u'') = \frac{\tan v'' + \tan u''}{1 - \tan v'' \tan u''} = \frac{v' + u'}{1 + \left(\frac{r}{R_s} - 1\right) e^{r/R_s}} \underset{r>0}{\Longrightarrow} |v''+u''| < \frac{\pi}{2}$$

Penrose diagram of Schwarzschild Black Hole

$$ds^2 = \frac{4R_s^3}{r} e^{-r/R_s} \frac{dv'' du''}{\cos^2 v'' \cos^2 u''} + r^2 d\Omega^2 , \qquad \tan u'' \tan v'' = -\left(\frac{r}{R_s} - 1\right) e^{r/R_s}$$

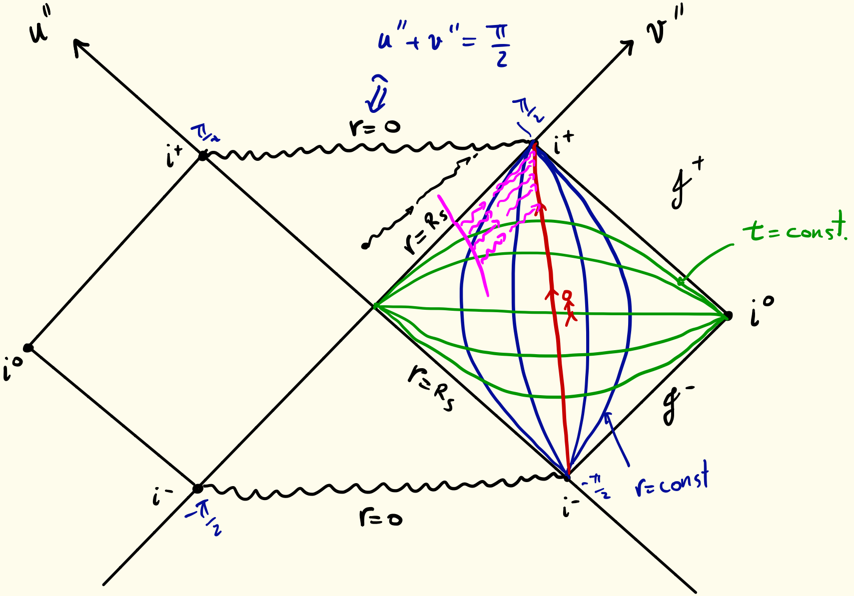
$$-\frac{\pi}{2} < u'', v'' < \frac{\pi}{2}$$

$$|u'' + v''| < \frac{\pi}{2}$$



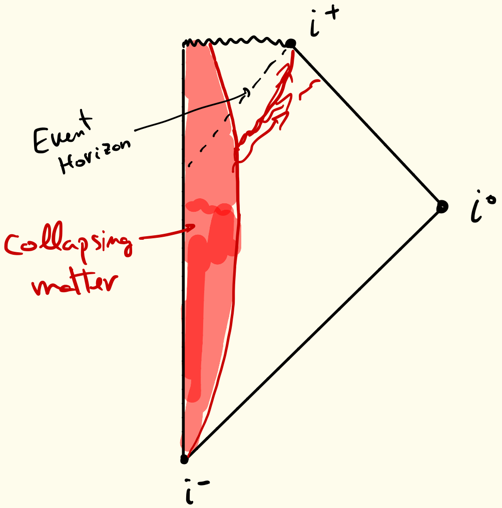
Penrose diagram of Schwarzschild Black Hole

$$ds^2 = \frac{4R_s^3}{r} e^{-r/R_s} \frac{dv'' du''}{\cos^2 v'' \cos^2 u''} + r^2 d\Omega^2 , \qquad \tan u'' \tan v'' = -\left(\frac{r}{R_s} - 1\right) e^{r/R_s}$$



Stars and Black Holes

The Penrose diagram of a BH formed by gravitational collapse is different from the one of an eternal BH :



Wisdom of the day

Death may be the greatest of all human blessings - Socrates

Lecture 12

More general Black Holes

Recap

Where were we?

General Relativity

Actions

Einstein equations

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Geodesic equations

$$\boxed{\frac{dx^\nu}{d\tau} \nabla_\nu \frac{dx^\mu}{d\tau} = 0} \longleftarrow \boxed{S = -m \int d\tau}$$

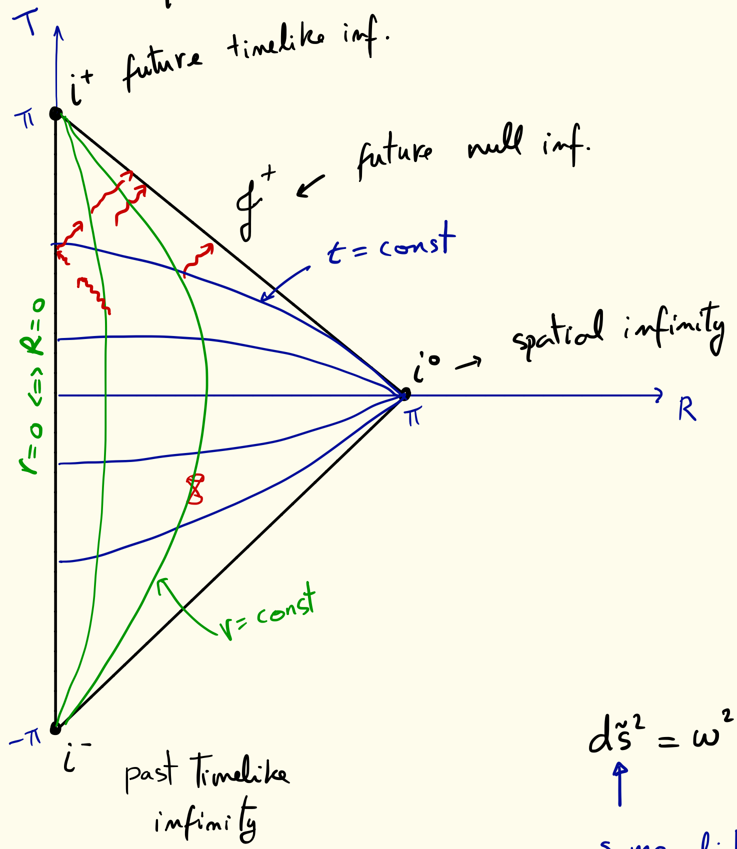
Energy-momentum tensor

Cosmological constant

$$\frac{\Lambda}{8\pi G} = \rho_{vac}$$

$$\boxed{T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta S_M}{\delta g^{\mu\nu}} - \frac{\Lambda}{8\pi G} g_{\mu\nu}}$$

Penrose diagram of Minkowski spacetime

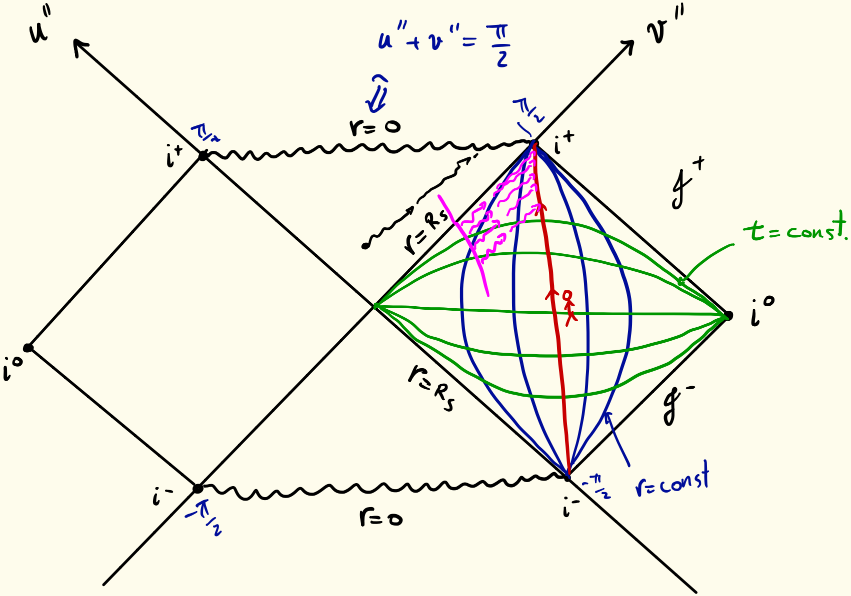


$$d\tilde{s}^2 = \omega^2 \, ds^2$$

same light cones

Penrose diagram of Schwarzschild Black Hole

$$ds^2 = \frac{4R_s^3}{r} e^{-r/R_s} \frac{dv'' du''}{\cos^2 v'' \cos^2 u''} + r^2 d\Omega^2 , \qquad \tan u'' \tan v'' = -\left(\frac{r}{R_s} - 1\right) e^{r/R_s}$$

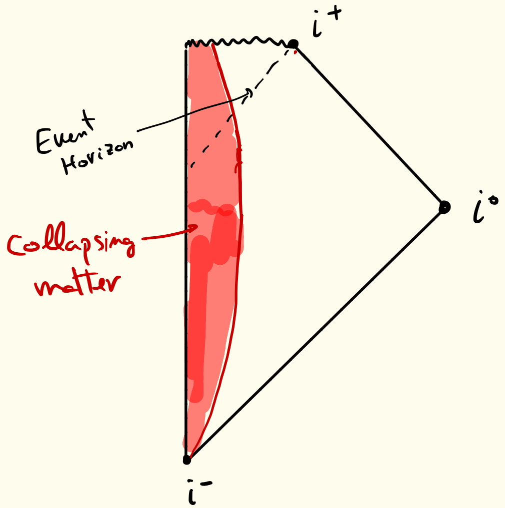


Plan for today

- Stars and black holes
- Charged and rotating black holes
- Some classical theorems
- BH thermodynamics

Stars and Black Holes

The Penrose diagram of a BH formed by gravitational collapse is different from the one of an eternal BH :



Spherically symmetric, static systems

$$ds^2 = \underbrace{-e^{2\alpha(r)}}_{g_{tt}} dt^2 + e^{2\beta(r)} dr^2 + r^2 d\Omega^2$$

Let's model the matter as a perfect fluid:

$$T_{\mu\nu} = (p+\rho)\, U_\mu U_\nu + p\, g_{\mu\nu}$$

static: $\quad U_\mu = (e^\alpha, 0, 0, 0)$ so that $\quad U_\mu U_\nu g^{\mu\nu} = -1.$

Unknowns: $\alpha(r), \beta(r), p(r), \rho(r)$

Exercise: Show that Einstein equations reduce to

(tt): $$\frac{1}{r^2} e^{-2\beta}\left(2r\beta' - 1 + e^{2\beta}\right) = 8\pi G \rho \qquad ' \equiv \frac{d}{dr}$$

(rr): $$\frac{1}{r^2} e^{-2\beta}\left(2r\alpha' + 1 - e^{2\beta}\right) = 8\pi G p$$

$(\theta\theta)$: $\Leftrightarrow (\phi\phi)$ $$e^{-2\beta}\left(\alpha'' + \alpha'^2 - \alpha'\beta' + \frac{\alpha' - \beta'}{r}\right) = 8\pi G p$$

Notice that

(tt) $\Leftrightarrow$ $$\Big(\underbrace{r - r e^{-2\beta}}_{\equiv\, 2Gm(r)}\Big)' = 8\pi G r^2 \rho \Leftrightarrow m'(r) = 4\pi r^2 \rho(r)$$

$$\Updownarrow$$

$$e^{-2\beta(r)} = 1 - \frac{2Gm(r)}{r}$$

$$\Leftrightarrow \boxed{m(r) = \int_0^r 4\pi \tilde{r}^2 \rho(\tilde{r})\, d\tilde{r}}$$

(rr) : $$\frac{d\alpha}{dr} = \frac{G\,m(r) + 4\pi\, G r^3 p(r)}{r\,[r - 2G\,m(r)]}$$

Instead of $(\theta\theta)$ we can use $\nabla_\mu T^{\mu\nu} = 0$:

$$\boxed{(\rho + p)\frac{d\alpha}{dr} = -\frac{dp}{dr}} \qquad \circledast$$

$$\Rightarrow \quad \boxed{\frac{dp}{dr} = -\frac{(\rho + p)\,[G\,m(r) + 4\pi\, G r^3 p(r)]}{r\,[r - 2G\,m(r)]}}$$

Tolman - Oppenheimer - Volkoff (TOV) equation

3 ind. E.E. + equation of state $\longrightarrow$ α, β, p, ρ

$p = p(\rho)$

$\beta \Updownarrow m$

Simplest model : incompressible fluid

$$\rho(r) = \begin{cases} \rho_* & , r<R \\ 0 & , r>R \end{cases} \Rightarrow \quad m(r) = \begin{cases} \frac{4}{3}\pi r^3 \rho_* & , r<R \\ M = \frac{4}{3}\pi R^3 \rho_* & , r>R \end{cases}$$

TOV eq $\underset{\text{Check}}{\Rightarrow}$ $$p(r) = \rho_* \frac{\sqrt{R^3 - 2GMr^2} - \sqrt{R^3 - 2GMR^2}}{3\sqrt{R^3 - 2GMR^2} - \sqrt{R^3 - 2GMr^2}} \quad , r<R$$

$\circledast \Rightarrow$ $$e^{\alpha} = \begin{cases} \frac{3}{2}\left(1 - \frac{2GM}{R}\right)^{1/2} - \frac{1}{2}\left(1 - \frac{2GMr^2}{R^3}\right)^{1/2} & , r<R \\ \left(1 - \frac{2GM}{r}\right)^{1/2} & , r>R \end{cases}$$

Notice that $p(r=R) = 0$ and $p(0) = \rho_* \dfrac{\sqrt{R} - \sqrt{R-2GM}}{3\sqrt{R-2GM} - \sqrt{R}} > 0 \Rightarrow R > \frac{9}{4}GM$

Buchdahl's theorem : $M < \dfrac{4R}{9G}$ for static system with no horizon.

Possible static astrophysical objects : $\left[P \sim \frac{GM^2}{R} \frac{1}{R^3} \sim \frac{1}{M_p^2} \rho^2 R^2 \right]$

- planets, asteroids

$$\rho \sim m_n (m_e \alpha)^3$$
$$P \sim m_e^4 \alpha^5$$
$$\Rightarrow \begin{cases} R \lesssim \frac{1}{\alpha} \frac{M_p}{m_n m_e} \\ M \lesssim m_n \frac{\alpha^{3/2} M_p^3}{m_n^3} \end{cases}$$

- white dwarf

$$\rho \sim m_n m_e^3$$
$$P \sim \frac{E = m_e}{\mathrm{vol} = \left(\frac{1}{m_e}\right)^3} \sim m_e^4$$
$$\Rightarrow \begin{cases} R \sim \frac{M_p}{m_n m_e} \\ M \sim \rho R^3 \sim m_n \left(\frac{M_p}{m_n}\right)^3 \end{cases}$$

- Neutron star

$$\rho \sim m_n^4$$
$$P \sim m_n^4$$
$$\Rightarrow \begin{cases} R \sim \frac{M_p}{m_n^2} \\ M \sim m_n \left(\frac{M_p}{m_n}\right)^3 \end{cases}$$

- ??

- Black hole

$$R \sim \frac{M}{M_p^2}$$

$$M \sim \begin{cases} M_\odot & \text{stellar collapse} \\ (10^6 - 10^9)\, M_\odot & \text{center of galaxies} \end{cases}$$

Wisdom of the day

Gustav Mahler

Special session by Louis Christoph

More general Black Holes

Charged Black Holes (Reissner – Nordström)

Static and spherically symmetric:

$$ds^2 = -\Delta\, dt^2 + \frac{dr^2}{\Delta} + r^2\, d\Omega^2 \quad , \quad \Delta = 1 - \frac{2GM}{r} + \frac{G(Q^2+P^2)}{r^2}$$

$$F = \frac{1}{2} F_{\mu\nu}\, dx^\mu \wedge dx^\nu = \frac{Q}{r^2}\, dt \wedge dr + P \sin\theta\, d\theta \wedge d\phi$$

electric charge

magnetic charge

[Check that

$F \leftrightarrow *F \iff Q \leftrightarrow P$]

Rotating Black Holes (Kerr)

$$ds^2 = -\left(1 - \frac{2GMr}{\rho^2}\right)dt^2 - \frac{4GMar\sin^2\theta}{\rho^2}\,dt\,d\phi + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)^2 - a^2\Delta\sin^2\theta\right]d\phi^2$$

$$\Delta(r) = r^2 - 2GMr + a^2 \quad , \quad \rho^2 = r^2 + a^2\cos\theta$$

$a = \frac{J}{M}$ ← angular momentum

Only two Killing vectors : ∂_t and ∂_ϕ

→ stationary and axisymmetric

Charged Rotating Black Holes (Kerr - Newman)

$$ds^2 = -\left(1 - \frac{\alpha}{\rho^2}\right)dt^2 - \frac{2\alpha a \sin^2\theta}{\rho^2}\, dt\, d\phi + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)^2 - a^2 \Delta \sin^2\theta\right] d\phi^2$$

$$\Delta(r) = r^2 - \alpha + a^2 \quad , \quad \rho^2 = r^2 + a^2 \cos\theta$$

$$a = \frac{J}{M}$$ ← angular momentum

$$\alpha = 2GMr + G(Q^2 + P^2)$$

4- vector potential :

$$A_t = \frac{Qr - Pa\cos\theta}{\rho^2} \quad , \quad A_\phi = \frac{-Qar\sin^2\theta + P(r^2+a^2)\cos\theta}{\rho^2}$$

$$A_r = 0 \quad , \quad A_\theta = 0$$

$$\{M, Q, P, J\}$$

More general Black Holes

No hair theorem : stationary, asymptotically flat black hole solutions to GR coupled to electromagnetism that are non-singular outside the event horizon are fully characterized by their mass, electric and magnetic charge, and angular momentum. $\{M, Q, P, J\}$

$\Rightarrow$ Information loss (paradox)

How to identify the event horizon?

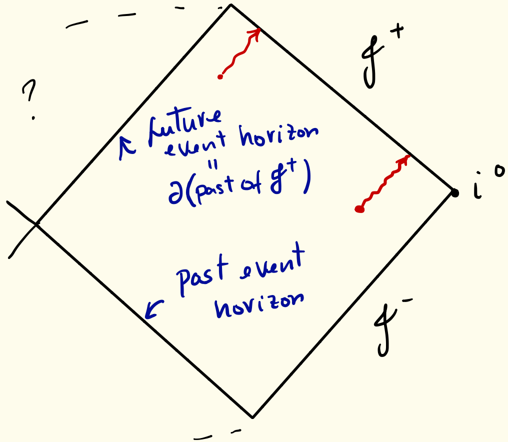
Assuming

- Stationary metric : $\partial_t g_{\mu\nu} = 0$

- asymptotically flat

$$ds^2 \xrightarrow[r\to\infty]{} -dt^2 + dr^2 + r^2 d\Omega^2$$

- $r=$ const surface becomes null at $r = r_H$ $\forall(\theta,\phi)$



Then $r = r_H$ is the horizon

$\partial_\mu r$ is normal to $r=$ const. surface $\rightarrow$ $\partial_\mu r$ is null $\Longleftrightarrow$

$$\boxed{g^{rr}(r=r_H) = 0}$$

Charged Black Holes

$$ds^2 = -\Delta\, dt^2 + \frac{dr^2}{\Delta} + r^2\, d\Omega^2 \quad , \quad \Delta = 1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}$$

$$F = \frac{Q}{r^2}\, dt \wedge dr$$

$$\boxed{g^{rr}(r = r_H) = 0}$$

$$\Downarrow$$

$$\Delta(r = r_H) = 0$$

$$\Downarrow$$

outer horizon

$$r = GM \pm \sqrt{G^2M^2 - GQ^2} \equiv r_{\pm}$$

inner horizon

Charged Black Holes

$$ds^2 = -\Delta\, dt^2 + \frac{dr^2}{\Delta} + r^2\, d\Omega^2 \quad , \quad \Delta = 1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}$$

$$F = \frac{Q}{r^2}\, dt \wedge dr$$

$$\boxed{g^{rr}(r = r_H) = 0}$$

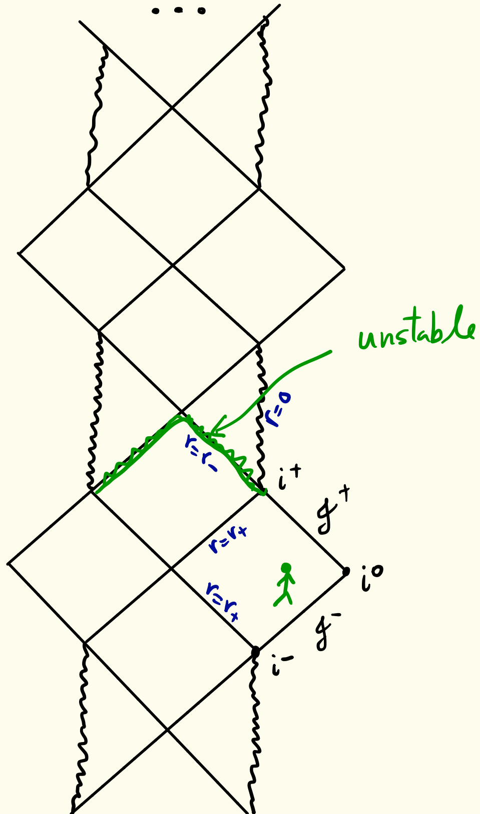
$$\Downarrow$$

outer horizon

$$r = GM \pm \sqrt{G^2M^2 - GQ^2} \equiv r_\pm$$

inner horizon

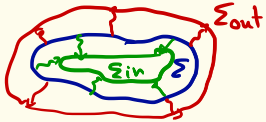
Penrose diagram for $GM^2 > Q^2$:



Do black holes actually form through gravitational collapse?

Probably yes because

- Singularity theorems:



$\text{area}(\Sigma_{out}) < \text{area}(\Sigma)$ → trapped
$\text{area}(\Sigma_{in}) < \text{area}(\Sigma)$

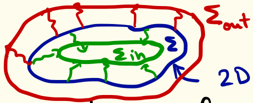
trapped surface
\+
strong energy cond. $\Rightarrow$ singularity

$$\left[T_{\mu\nu} t^\mu t^\nu \geq \frac{1}{2} T_\lambda{}^\lambda t^\sigma t_\sigma \quad \forall t^\mu \text{ timelike} \right]$$

incomplete timelike geodesic

Do black holes actually form through gravitational collapse?

Probably yes because:



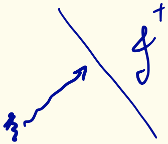
area(Σ) > area(Σ_{out})

2D spatial surface

- Singularity Theorems: trapped surface + strong energy condition $\Rightarrow$ singularity

$$\left[T_{\mu\nu} t^\mu t^\nu \geq \frac{1}{2} T_\lambda^{\ \lambda} t^\sigma t_\sigma \quad \forall t^\mu \text{ timelike} \right]$$

~~[illegible]~~ incomplete timelike geodesic



- Cosmic censorship conjecture:

Naked singularities cannot form in gravitational collapse from generic, initially nonsingular states in an asymptotically flat spacetime obeying the dominant energy condition.

Hawking's area theorem :

- Weak energy condition
  $[T_{\mu\nu} t^\mu t^\nu \geq 0]$
- Cosmic censorship
- asymptotic flatness

$\Longrightarrow$ Area of future event horizon is non-decreasing

$$\boxed{dA \geq 0}$$